Math 146C - Ordinary and Partial Differential Equations III
Quiz 3
April 21, 2011

Name: Key

1	2	Total
/7	/3	/10

Problem 1. *(7 points) Find the solution of the heat conduction problem:*

$$\begin{cases} 100u_{xx} = u_t, & 0 < x < 1,\ t > 0 \\ u(0,t) = 0,\ u(1,t) = 0, & t > 0; \\ u(x,0) = \sin 2\pi x - \sin 5\pi x, & 0 \le x \le 1 \end{cases}$$

Problem 2. *(3 points) Let $\sum_{n=1}^{\infty} x_n$ be a convergent series. Prove or disprove the following statement:*

$$\left(\sum_{n=1}^{\infty} x_n\right)^2 = \sum_{n=1}^{\infty} x_n^2.$$

① Here $\alpha^2 = 100$ and $L = 1$, so

$$u(x,t) = \sum_{n=1}^{\infty} c_n e^{-100n^2\pi^2 t} \sin n\pi x \text{ where}$$

$$c_n = \frac{2}{1}\int_0^1 u(x,0) \sin n\pi x \, dx = \begin{cases} 1, & n=2 \\ -1, & n=5 \\ 0, & \text{o.w.} \end{cases}$$

Thus $u(x,t) = e^{-400\pi^2 t} \sin 2\pi x - e^{-2500\pi^2 t} \sin 5\pi x$.

② Let $x_n = \frac{(-1)^n}{\sqrt{n}}$, by the alternating series test, $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges, so $\left(\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}\right)^2 < \infty$. However,

$\sum_{n=1}^{\infty} x_n^2 = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges $\therefore \left(\sum_{n=1}^{\infty} x_n\right)^2 \neq \sum_{n=1}^{\infty} x_n^2$.